Categorical independent variables and spatial regression: interpretation and reporting

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Introduction

Introduction

- While researchers have addressed some of the challenges raised by limited dependent variables in spatial regression, the interpretation of independent categorical variables has not been given much attention; this issue came up in discussion with James LeSage during a workshop in Vienna at ERSA in 2016.
- This question appears when the spatial lag of an independent variable is included, as would typically be the case in what are known as Durbin models.
- The spatial lag is usually taken as the mean of the values of the variable at neighbouring observations for row-standardised spatial weights, or the sum for binary spatial weights, neither of which permit intuitive interpretation for categorical variables.
- Slides and source at: https://rsbivand.github.io/eqc25_talk

Pivot to causality

- In their critical appraisal of spatial econometrics, Gibbons and Overman (2012) state that, with regard to causality, "[t]hese questions are fundamentally of the type 'if we change x, what do we expect to happen to y?' "
- In spatial autoregressive models using the average of neighbours' values of the dependent variable as an explanatory variable, the coefficient of the spatially lagged dependent variable interacts with the coefficients of the independent variables (Kelejian, Tavlas, and Hondroyiannis 2006; LeSage and Fischer 2008; Ward and Gleditsch 2008; LeSage and Pace 2009).

The spatial lag model

- A model with a spatial process in the dependent variable is termed a spatial lag model (SLM, often SAR - spatial autoregressive) (LeSage and Pace 2009)

$$\mathbf{y} = \rho_{\mathrm{Lag}} \mathbf{W} \mathbf{y} + \mathbf{X} \beta + \varepsilon,$$

- where ρ_{Lag} is a scalar spatial parameter, $\mathbf{W}$ is an $n \times n$ spatial weights matrix,
- $\mathbf{y}$ is an $(n \times 1)$ vector of observations on a response variable taken at each of n locations, $\mathbf{X}$ is an $(n \times k)$ matrix of covariates usually including the intercept, β is a $(k \times 1)$ vector of parameters, and ε is an $(n \times 1)$ vector of independent and identically distributed disturbances.

Impacts in the spatial lag model

- In the spatial lag model, the change in $\mathbf{y}$ caused by a change in $\mathbf{x}_r$, is taken as $\partial y_i / \partial x_{jr} = ((\mathbf{I} - \rho_{\text{Lag}} \mathbf{W})^{-1} \mathbf{I} \beta_r)_{ij}$,
- where $\mathbf{I}$ is the $n \times n$ identity matrix, $i, j = 1, ..., n$, y_i is the value of the dependent variable at i and x_{jr} is the value of the r-th independent variable at j.
- What is ∂x_{jr} when $\mathbf{x}_r$ is a categorical variable?

Spatial Durbin

- Work reviewed by Mur and Angulo (2006) on the Durbin model; the Durbin model adds the spatially lagged covariates to the covariates included in the spatial lag model, giving a spatial Durbin model (SDM) with different processes in the response and covariates.

$$\mathbf{y} = \rho_{\mathrm{Lag}} \mathbf{W} \mathbf{y} + \mathbf{X} \beta + \mathbf{W} \mathbf{X} \gamma + \varepsilon,$$

- where γ is a $(k' \times 1)$ vector of parameters.
- k' defines the subset of the intercept and covariates, often $k' = k - 1$ when using row standardised spatial weights and omitting the spatially lagged intercept.
- The SLX model omits the $\rho_{\mathrm{Lag}} \mathbf{W} \mathbf{y}$ term; the GNM model includes an additional spatial process in the residuals.

Impacts in the spatial lag and Durbin model

- In the spatial lag model, the impacts are taken from this matrix: $S_r(\mathbf{W}) = ((\mathbf{I} - \rho_{\mathrm{Lag}}\mathbf{W})^{-1}\mathbf{I}\beta_r)$. Direct impacts are the mean of the principal diagonal of S_r
- In the spatial Durbin model, spatially lagged independent variables enter impacts through γ: $S_r(\mathbf{W}) = ((\mathbf{I} - \rho_{\mathrm{Lag}}\mathbf{W})^{-1}(\mathbf{I}\beta_r - \mathbf{W}\gamma_r))$
- In the SLX model, β_r are direct and γ_r indirect impacts
- What is $\mathbf{W}\mathbf{x}_r$, and hence γ_r, computed when the r-th independent variable is categorical?

Interpreting spatial (econometrics) model

- Arel-Bundock, Greifer, and Heiss (2024), Arel-Bundock (2025a) and others have been working hard to improve how statistical models are interpreted.
- For categorical variables, it is important to report the comparison reflected in the way the variable is organised
- Equally, it is important to avoid pretending that categorical (and bounded) independent variables are unbounded and observed on a continuum
- There are several open issues in the Github repository of `spatialreg` (Bivand and Piras 2025), an R package I maintain, concerning the handling of categorical independent variables.

Garbage data set

Garbage data set i

- To explore how to begin handling categorical independent variables, we shall be using a data set from Bivand and Szymanski (1997) and Bivand and Szymanski (2000), extending results in Szymanski and Wilkins (1993), Szymanski (1996) and Bello and Szymanski (1996).
- The question posed in the research reported in these articles is whether the real net costs of garbage collection in English local authority districts (which had a statutory obligation to collect garbage) were reduced when compulsory competitive tendering was introduced from the late 1980s to the early 1990s.
- Not all districts introduced compulsory competitive tendering at the same time, and some had not begun when data collection ceased. Of 366 districts, only 324 had completed at this point, and the data used relate to these districts.

Garbage data set ii

- Since compulsory competitive tendering was implemented in different years, the data set reports real district net expenditure on garbage collection for the year before the implementation of compulsory competitive tendering (pre-CCT), and the year after (post-CCT).
- As mentioned in Bivand and Szymanski (2000), the observations are agents, their boundaries do not change during the study period, and as entities they meet reasonable behavioural expectations.
- Revelli (2003), Brueckner (2003), Revelli (2006), Revelli and Tovmo (2007) and others engage with a literature including Case, Hines, and Rosen (1993) and Besley and Case (1995) concerning strategic interactions, of which yardstick competition may play a part.

Garbage data set iii

- In Bivand and Szymanski (1997), the identification of residual spatial autocorrelation in the results given by Bello and Szymanski (1996) was used to develop a spatial yardstick competition framework, in which districts without compulsory competitive tendering were more likely to be influenced by the behaviour of their proximate neighbours than by general market conditions.
- The data used in Bivand and Szymanski (2000) have been matched with simplified boundaries for English districts, and may be read in the usual way.

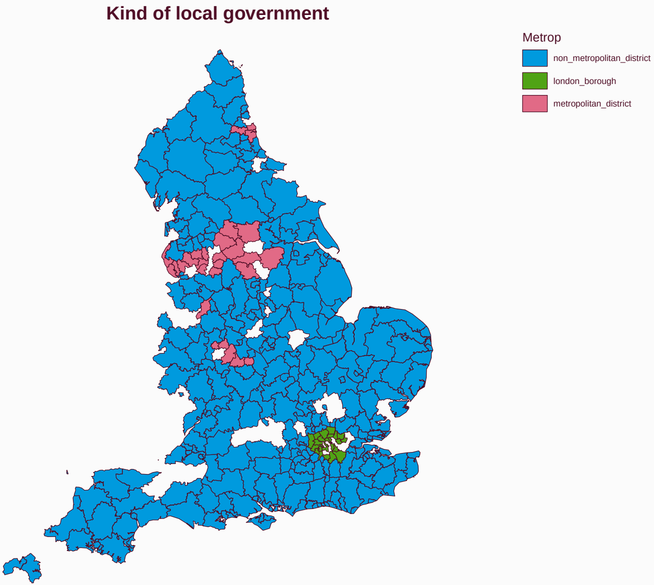
Categorical variables and garbage

- In Bivand and Szymanski (2000), two categorical variables are used in accounting for the real net cost of garbage collection and disposal:
- `Metrop` for the kind of local government authority: non-metropolitan district, metropolitan district, or London borough, and
- `Majority` for the political party controlling the local government authority: no overall control, Conservative, or Labour.
- For warnings about spatially lagged categorical variables to work, the variables should be set as `"factor"` before tests are run or models fitted: here for example `Metrop` is read as `"character"` and needs to be coerced to `"factor"`:

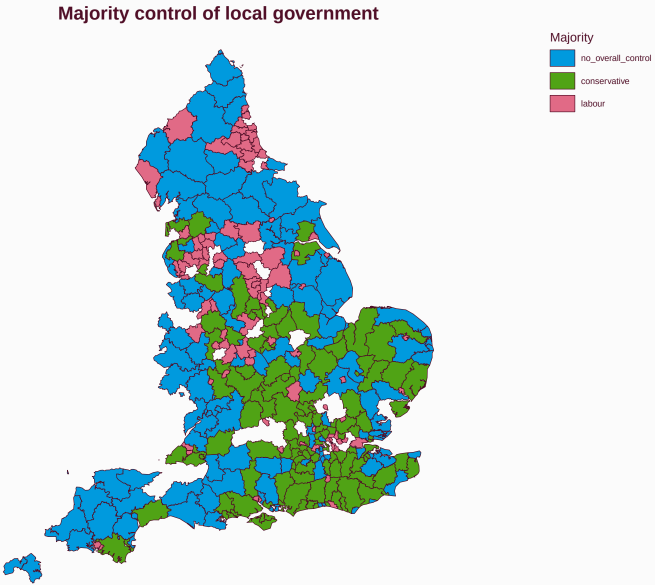
Categorical variables and garbage

```r
> library(sf)
> eng324 <- st_read("data/eng324s.gpkg.zip", quiet=TRUE)
> unname(attr(attr(model.frame(~ Metrop, eng324), "terms"), "dataClasses"))
## [1] "character"
> eng324$Metrop <- factor(eng324$Metrop)
> levels(eng324$Metrop) <- c("non_metropolitan_district", "london_borough",
+  "metropolitan_district")
> unname(attr(attr(model.frame(~ Metrop, eng324), "terms"), "dataClasses"))
## [1] "factor"
> eng324$Majority <- factor(eng324$Majority)
> levels(eng324$Majority) <- c("no_overall_control", "conservative", "labour")
```

Kind of local government (`mapsf` package)



Majority control of local government (`mapsf` package)



Contrasts

- Use of categorical variables on the right-hand side of regression models involves the creation of blocks of “dummy” 0/1 variables representing the category levels.
- Typically, the first category is taken as the reference category (and becomes part of the intercept), and the remaining categories are included as dummy variables in relation to the reference category.
- The default contrast for unordered factors in R is `"contr.treatment"`; if a different category is needed as reference category, this may be changed using `relevel()`.
- The contrasts are contrasts of the succeeding category means with the mean of the reference category.

Factor contrasts

```
> attr(C(eng324$Metrop), "contrasts")
##          unordered
## "contr.treatment"
> contrasts(eng324$Metrop)
##                            london_borough metropolitan_district
## non_metropolitan_district               0                     0
## london_borough                          1                     0
## metropolitan_district                   0                     1
> mm0 <- model.matrix(~ Metrop - 1, eng324)
> apply(mm0, 2, sum)
## Metropnon_metropolitan_district            Metroplondon_borough     Metropmetropolitan_district
##                             272                              24                              28
```

Is `house` continuous?

- The `codingMatrices` package (Venables 2023) suggests alternatives for ordered factors (ordinal variables).
- Bivand and Szymanski (2000) also use a `house` variable, which is the proportion of garbage pick-up points that are residential, rather than business.
- Only one district — City of London — has a low proportion, and a number of others, mostly tourist destinations, have proportions under 0.9; consequently, the variable might better be represented as an ordered factor.

house as factor

```r
> fhouse <- cut(eng324$house, c(0.0, 0.25, 0.90, 1))
> table(fhouse)
## fhouse
##   (0,0.25] (0.25,0.9]    (0.9,1]
##          1         23        300
> contrasts(fhouse)
##            (0.25,0.9] (0.9,1]
## (0,0.25]            0       0
## (0.25,0.9]          1       0
## (0.9,1]             0       1
> attr(C(fhouse), "contrasts")
##          unordered
## "contr.treatment"
```

house as ordered

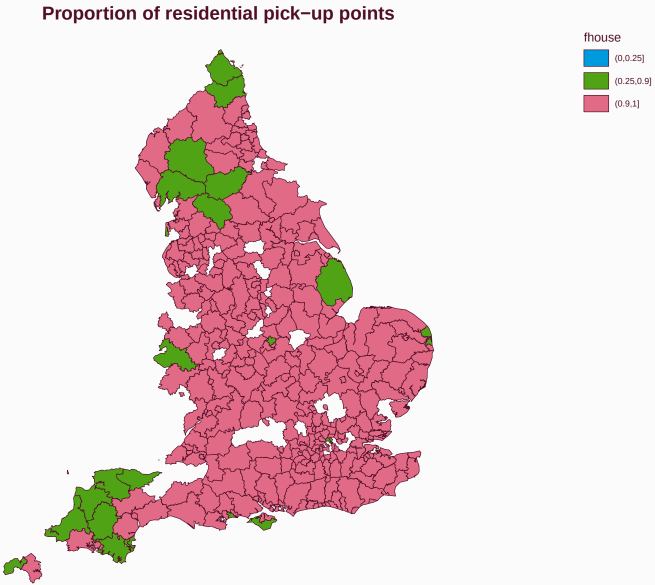
```
> fhouse <- as.ordered(fhouse)
> contrasts(fhouse)
##                 .L         .Q
## [1,] -7.071068e-01  0.4082483
## [2,] -7.850462e-17 -0.8164966
## [3,]  7.071068e-01  0.4082483
> attr(C(fhouse), "contrasts")
##      ordered
## "contr.poly"
```

`house` ordered contrasts

- The default contrast for ordered factors is `"contr.poly"`, which uses a polynomial in the numerical values of the levels, here linear `.L` and quadratic `.Q`, which assumes that the levels are equally spaced.
- `codingMatrices` suggests using `contr.diff` instead, operating like `"contr.treatment"`; the contrasts are then contrasts of the succeeding category mean with the mean of the preceding category.

```
> library(codingMatrices)
> contrasts(fhouse) <- "contr.diff"
> attr(fhouse, "contrasts")
## [1] "contr.diff"
> contrasts(fhouse)
##   m2-m1 m3-m2
## 1     0     0
## 2     1     0
## 3     1     1
> eng324$fhouse <- fhouse
```

Proportion of residential pick-up points (**`mapsf`** package)



Connecting the district contiguities

- As can be seen from the maps, the exclusion of districts that had not reported adopting competitive tendering in the data collection period creates gaps.
- One subgraph is expected (Isle of Wight with two districts), but western Cornwall is not.
- Linking the two subgraphs and three singletons to the main graph step-by-step using `sf::st_distance` to the nearest proximate polygon boundaries removes the problem.

Connecting the district contiguities (`mapsf` package)

Diagnostics and interpretation

Original final model

- In Bivand and Szymanski (2000), we concluded that the log real net cost of garbage collection could be modelled using the log of the number of pick-up points, the proportion of residential pick-up points, the log density of the count of pick-up points in the local authority area, the type of district factor, the log real wage level, and the district political control factor.
- We also found that, when the district political control factor was included, and the spatial lag model was used (including the spatial lag of log real net cost of garbage collection on the right-hand side), observed neighbour costs were highly significant pre-CCT, but not post-CCT.
- However, log density and log local wage levels became highly significant post-CCT, meaning that instead of local yardstick competition affecting costs, real drivers (distances between pick-up points, wage costs) affect costs. We'll use the ordinal version of the proportion of residential pick-up points.

OLS models

```r
> form_pre_majf <- log(realNetPre) ~ log(units) + fhouse + log(dens) + Metrop +
+  Majority + log(realWgPre)
> form_post_majf <- update(form_pre_majf,
+  log(realNetPst) ~ . + log(realWgPst) - log(realWgPre))
> pre_majf_lm <- lm(form_pre_majf, eng324)
> post_majf_lm <- lm(form_post_majf, eng324)
```

Tabulated output uses the **`modelsummary`** package (Arel-Bundock 2025b)

OLS models

	Pre-CCT OLS	Post-CCT OLS
(Intercept)	-3.561**	-5.689***
units [log]	0.971***	0.939***
fhousem2-m1	-1.607***	-1.166***
fhousem3-m2	0.046	-0.041
dens [log]	-0.023+	-0.052***
Metrop [london_borough]	0.190**	0.108+
Metrop [metropolitan_district]	0.023	0.035
Majority [conservative]	0.043	-0.012
Majority [labour]	0.208***	0.193***
realWgPre [log]	0.297	
realWgPst [log]		0.710***

+ p <0.1, * p <0.05, ** p <0.01, *** p <0.001

OLS models and spatially autocorrelated residuals

- Pre-CCT OLS shows spatially autocorrelated residuals with a Moran value of 0.11432 with a p-value of 0.0006863
- Post-CCT OLS shows spatially autocorrelated residuals with a reduced Moran value of 0.09745 with a p-value of 0.0028028
- Lagrange multiplier tests are less significant for Post-CCT than Pre-CCT, neither clearly indicate the appropriate kind of spatial model

Are spatially lagged independent variables relevant?

- It has been widely argued that adding spatially lagged independent variables can make models robust to a range of possible mis-specifications
- If the entity boundaries are arbitrary, or not designed for the data collection process, or if spatial processes in the dependent variable and/or the residuals are unclear, such a step may be appealing
- From Elhorst (2010) and Halleck Vega and Elhorst (2015), it has been seen as advisable to start from the General Nested Model (GNM) eliminating unneeded terms
- Tests for omitted spatially lagged independent variables (Koley and Bera 2024; Koley 2024) suggest that Durbin terms $\mathbf{WX}$ should be added for both pre-CCT and post-CCT models

Spatial Durbin tests: pre-CCT with factors

```
> pre_SD0 <- summary(spdep::SD.RStests(pre_majf_lm, listw=lwW))
## Warning in warn_factor_preds(have_factor_preds): use of spatially lagged factors (categorical variable
## fhouse, Metrop, Majority
## is not well-understood
> pre_SD0
##  Rao's score test spatial Durbin diagnostics
## data:
## model: lm(formula = form_pre_majf, data = eng324)
## weights: lwW
##
##               statistic parameter   p.value
## SDM_RSlag        8.1418         1 0.0043257 **
## SDM_adjRSlag     8.1720         1 0.0042543 **
## SDM_RSWX        27.9965         9 0.0009552 ***
## SDM_adjRSWX     28.0267         9 0.0009441 ***
## SDM_Joint       36.1685        10 7.875e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Alerting the user to spatially lagged categorical variables

- If we just accept that $\mathbf{WX}$ are included (omitting the intercept if $\mathbf{W}$ is row-standardised), we should be warned if this creates spatially lagged categorical variables
- This warning has been introduced into `SD.RStests` in `spdep` (Bivand 2025), and model fitting functions in `spatialreg` (Bivand and Piras 2025) for models including $\mathbf{WX}$
- The `Durbin=` argument takes a formula object, so permits the omission of categorical variables

Spatial Durbin tests

	Pre-CCT	Pre-CCT w/o factors	Post-CCT	Post-CCT w/o factors
SDM_RSlag	8.142**	8.142**	4.831*	4.831*
SDM_adjRSlag	8.172**	10.243**	5.938*	9.017**
SDM_RSWX	27.996***	16.687***	32.848***	15.618**
SDM_adjRSWX	28.027***	18.788***	33.955***	19.804***
SDM_Joint	36.168***	26.930***	38.786***	24.635***

+ p <0.1, * p <0.05, ** p <0.01, *** p <0.001

Spatially lagged categorical variables

In practice, the values are weighted averages across neighbours of the dummy variables initially constructed from the factor by `model.matrix`, so no longer `0/1` dummy variables, where the non-binary values imply some undefined mixture of categories. Using `mm0` constructed above from `Metrop`:

```
> sum(mm0 > 0 & mm0 < 1)
## [1] 0
> length(unique(c(mm0)))
## [1] 2
> W <- spdep::listw2mat(lwW)
> mm1 <- W %*% mm0
> sum(mm1 > 0 & mm1 < 1)
## [1] 193
> length(unique(c(mm1)))
## [1] 28
```

SLX models

Adding $\mathbf{WX}$ to the existing OLS model lets us use Lagrange multiplier tests (following Koley (2024)) to test for remaining residual spatial autocorrelation. First, the models, showing that the warning for poorly understood spatially lagged categorical variables is shown where needed:

```
> library(spatialreg)
> pre_majf_slx <- lmSLX(form_pre_majf, eng324, listw=lwW)
## Warning in warn_factor_preds(have_factor_preds): use of spatially lagged factors (categorical variable
## fhouse, Metrop, Majority
## is not well-understood
> pre_majf_slxD <- lmSLX(form_pre_majf, eng324, listw=lwW,
+  Durbin=update(form_pre_majf, ~ . - Metrop - Majority - fhouse))
> post_majf_slx <- lmSLX(form_post_majf, eng324, listw=lwW)
## Warning in warn_factor_preds(have_factor_preds): use of spatially lagged factors (categorical variable
## fhouse, Metrop, Majority
## is not well-understood
> post_majf_slxD <- lmSLX(form_post_majf, eng324, listw=lwW,
+  Durbin=update(form_post_majf, ~ . - Metrop - Majority - fhouse))
```

Diagnostic tests for SLX models and spatially lagged categorical variables

	Pre-CCT SLX	Pre-CCT SLX w/o factors	Post-CCT SLX	Post-CCT SLX w/o factors
GNM_RSerr	2.568	6.400*	4.248*	5.132*
GNM_RSlag	5.285*	9.197**	4.952*	8.224**
GNM_adjRSerr	13.806***	0.981	0.393	1.648
GNM_adjRSlag	16.523***	3.778+	1.097	4.740*
GNM_SARMA	19.091***	10.179**	5.345+	9.871**

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Impacts for SLX models

- In SLX models (and equivalently SDEM), average direct impacts are simply the coefficients of the independent variables, the change in y_i from a unit change in the r-th independent variable x_{ir} averaged over the i observations
- Average indirect impacts look at the change in y_i from a unit change in the r-th independent variable $\mathbf{Wx}_{r_i}$, often known as local impacts taken over the j neighbours of i
- Average total impacts are the sums of these two, total impacts and their standard errors are calculated using `multcomp::glht` (Hothorn, Bretz, and Westfall 2025)

Total impacts for SLX models and spatially lagged categorical variables

- When spatially lagged categorical variables are omitted using the `Durbin=` argument, average total impacts for those variables are equal to the average direct impacts
- For SLX and SDEM models, average indirect impacts are reported as `NA` - not available - for all variables omitted from the Durbin formula
- For SLM, SDM and GNM models, average indirect impacts for variables omitted from the Durbin formula are reported as the difference between average direct and average total impacts
- Factors are marked `"(F)"`, other variables `"dy/dx"` - should impacts' output also warn the user beyond this annotation? `"dy/dx"` makes sense where `"slope"` makes sense, what should we do when it doesn't?

Total impacts for SLX models and spatially lagged categorical variables

	Pre-CCT SLX	Pre-CCT SLX w/o factors	Post-CCT SLX	Post-CCT SLX w/o factors
log(units) dy/dx	0.817***	0.922***	0.752***	0.870***
fhousem2-m1 (F)	−1.205	−1.515***	−0.994	−1.084***
fhousem3-m2 (F)	−0.005	0.036	−0.062	−0.047
log(dens) dy/dx	0.028	0.048*	−0.015	0.015
Metroplondon_borough (F)	0.234*	0.076	0.235*	0.011
Metropmetropolitan_district (F)	0.229+	−0.029	0.300*	0.000
Majorityconservative (F)	0.067	0.016	0.043	−0.039
Majoritylabour (F)	0.262***	0.217***	0.257***	0.203***
log(realWgPre) dy/dx	0.188	−0.041		
log(realWgPst) dy/dx			0.633*	0.479+

+ p <0.1, * p <0.05, ** p <0.01, *** p <0.001

Reporting spatial lag and spatial Durbin models (and GNM)

- Because in SLM, SDM, SARAR/SAC and GNM, the coefficient of the spatially lagged dependent variable ρ_{Lag} interacts with β and γ if present, the regression coefficients cannot be interpreted simply
- Consequently, reporting should use average impacts, where average total impacts correspond to per-variable unit increments expressing $\partial y_i / \partial x_{jr}$ for the r-th variable
- Provisionally, the status of categorical variables in SLM and SARAR/SAC is probably less questionable, as $\mathbf{x}_r$ is the original dummy variable
- However, in SDM and GNM, spatially lagged categorical variables do enter, so model fitting generates warnings, and the `Durbin=` argument can be used to omit them from that term

Pre-CCT total impacts for SLM and spatially lagged categorical variables

	Pre-CCT SLM	Pre-CCT SDM	Pre-CCT SDM w/o factors
log(units) dy/dx	1.127^{***}	0.819^{***}	0.940^{***}
fhousem2-m1 (F)	-1.791^{***}	-1.326	-1.945^{***}
fhousem3-m2 (F)	0.021	-0.007	0.052
log(dens) dy/dx	-0.032^{*}	0.032	0.047^{+}
Metroplondon_borough (F)	0.116	0.230^{+}	0.059
Metropmetropolitan_district (F)	-0.067	0.229	-0.075
Majorityconservative (F)	0.036	0.076	0.017
Majoritylabour (F)	0.232^{***}	0.265^{**}	0.253^{***}
log(realWgPre) dy/dx	0.239	0.167	-0.057

+ p <0.1, * p <0.05, ** p <0.01, *** p <0.001

Post-CCT total impacts for SDM and spatially lagged categorical variables

	Post-CCT SLM	Post-CCT SDM	Post-CCT SDM w/o factors
log(units) dy/dx	1.054^{***}	0.738^{***}	0.878^{***}
fhousem2-m1 (F)	-1.252^{***}	-1.006	-1.353^{***}
fhousem3-m2 (F)	-0.064	-0.047	-0.044
log(dens) dy/dx	-0.060^{***}	-0.014	0.020
Metroplondon_borough (F)	0.055	0.262^{*}	-0.014
Metropmetropolitan_district (F)	-0.029	0.318^{*}	-0.043
Majorityconservative (F)	-0.022	0.046	-0.053
Majoritylabour (F)	0.208^{***}	0.267^{***}	0.233^{***}
log(realWgPst) dy/dx	0.693^{**}	0.615^{+}	0.468

+ p <0.1, * p <0.05, ** p <0.01, *** p <0.001

Model goodness-of-fit

- Without going all the way to GNM, we have four basic models: OLS, SLM, SLX and SDM with the latter two also having without factor variants to compare
- Of the many possible goodness-of fit measures available, the likelihood ratio (LR) is perhaps the simplest
- LR does not penalise the additional parameters required by including the spatially lagged independent variables
- On the other hand, the χ^2 probability values do use the count of included variables, so LR is quite robust

Pre-CCT likelihood ratio tests

	OLS	SLM	SLX w/o factors	SLX	SDM w/o factors
SLM	8.613**				
SLX w/o factors	17.132***	8.519*			
SLX	29.281***	20.667**	12.148+		
SDM w/o factors	26.688***	18.075***	9.556**	−2.592	
SDM	34.849***	26.236**	17.717*	5.568*	8.161

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Post-CCT likelihood ratio tests

	OLS	SLM	SLX w/o factors	SLX	SDM w/o factors
SLM	4.988^{*}				
SLX w/o factors	16.007^{**}	11.020^{**}			
SLX	34.636^{***}	29.648^{***}	18.628^{**}		
SDM w/o factors	24.668^{***}	19.680^{***}	8.660^{**}	-9.968^{+}	
SDM	39.771^{***}	34.784^{***}	23.764^{**}	5.136^{*}	15.103^{*}

+ p <0.1, * p <0.05, ** p <0.01, *** p <0.001

Model goodness-of-fit and interpretability

- If we follow the goodness-of-fit measures, we might end up with SDM w/o factors pre-CCT, and perhaps the same post-CCT
- However, ρ_{Lag} is 0.1323 in SLM pre-CCT, 0.1024 post-CCT; for SDM pre-CCT 0.1808, post-CCT 0.172, and w/o factors pre-CCT 0.2237, post-CCT 0.2144
- Including spatially lagged independent variables, even without factors, boosts spatial dependency in the model, rather than accounting for it
- Here, the districts are agents, and neighbours' pick-up point density or pick-up point count really do not affect costs
- The problem may come from the real wage variable, which is observed at an aggregated level and copied to districts, but omitting it changes little, so staying with SLM seems judicious

Discussion

Discussion

- It is hard to conclude now, but it seems reasonable to ask how spatial regression models should be interpreted
- We know that impacts matter, but the concepts now used in recent developments (Arel-Bundock, Greifer, and Heiss 2024; Arel-Bundock 2025a) and similar go further
- Attempting to examine just the handling of spatially lagged factors in SLX, SDEM, SDM and GNM models suggests that we are some way from being able to move towards interpretability
- But at least R software for spatial econometrics models and tests now has warnings where Durbin terms are used

sessionInfo I

```
> sessionInfo()

R version 4.5.1 (2025-06-13)
Platform: x86_64-pc-linux-gnu
Running under: Fedora Linux 42 (Workstation Edition)

Matrix products: default
BLAS/LAPACK: FlexiBLAS LIBFLEXIBLAS_OPENBLAS-SERIAL.SO;  LAPACK version 3.12.0

locale:
 [1] LC_CTYPE=en_GB.UTF-8       LC_NUMERIC=C               LC_TIME=en_GB.UTF-8
 [4] LC_COLLATE=en_GB.UTF-8     LC_MONETARY=en_GB.UTF-8    LC_MESSAGES=en_GB.UTF-8
 [7] LC_PAPER=en_GB.UTF-8       LC_NAME=C                  LC_ADDRESS=C
[10] LC_TELEPHONE=C             LC_MEASUREMENT=en_GB.UTF-8 LC_IDENTIFICATION=C

time zone: Europe/Oslo
tzcode source: system (glibc)

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
[1] spatialreg_1.4-2     Matrix_1.7-4         spData_2.3.4         modelsummary_2.5.0
```

sessionInfo ii

```
 [5] codingMatrices_0.4.0 mapsf_1.0.0          sf_1.0-21

loaded via a namespace (and not attached):
 [1] tidyselect_1.2.1    dplyr_1.1.4         fastmap_1.2.0       TH.data_1.1-4
 [5] bayestestR_0.17.0   digest_0.6.37       estimability_1.5.1  lifecycle_1.0.4
 [9] LearnBayes_2.15.1   survival_3.8-3      magrittr_2.0.3      compiler_4.5.1
[13] rlang_1.1.6         tools_4.5.1         igraph_2.1.4        yaml_2.3.10
[17] data.table_1.17.8   knitr_1.50          sp_2.2-0            classInt_0.4-11
[21] multcomp_1.4-28     KernSmooth_2.23-26  tinytable_0.13.0    purrr_1.1.0
[25] grid_4.5.1          datawizard_1.2.0    fansi_1.0.6         xtable_1.8-4
[29] e1071_1.7-16        future_1.67.0       globals_0.18.0      emmeans_1.11.2-8
[33] MASS_7.3-65         tinytex_0.57        insight_1.4.2       cli_3.6.5
[37] mvtnorm_1.3-3       rmarkdown_2.29      generics_0.1.4      future.apply_1.20.0
[41] performance_0.15.1  parameters_0.28.1   spdep_1.4-2         DBI_1.2.3
[45] proxy_0.4-27        splines_4.5.1       parallel_4.5.1      effectsize_1.0.1
[49] s2_1.1.9            vctrs_0.6.5         boot_1.3-32         sandwich_3.1-1
[53] jsonlite_2.0.0      litedown_0.7        listenv_0.9.1       maplegend_0.3.0
[57] spDataLarge_2.0.6   tidyr_1.3.1         units_0.8-7         glue_1.8.0
[61] parallelly_1.45.1   codetools_0.2-20    deldir_2.0-4        tables_0.9.31
[65] lmtest_0.9-40       fractional_0.1.3    tibble_3.3.0        pillar_1.11.0
[69] htmltools_0.5.8.1   R6_2.6.1            wk_0.9.4            evaluate_1.0.5
[73] lattice_0.22-7      backports_1.5.0     broom_1.0.9         class_7.3-23
[77] Rcpp_1.1.0          coda_0.19-4.1       nlme_3.1-168        checkmate_2.3.3
```

sessionInfo iii

```
[81] xfun_0.53           zoo_1.8-14         pkgconfig_2.0.3
```

Aftermatter

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